Licensing of a Drastic Innovation with Product Differentiation and Optimal Number of Licenses

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April 2004

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Abstract

I analyze licensing of a drastic innovation when products are differentiated due to consumer and/or product heterogeneity with two part licensing contracts. I show that an industry insider prefers to divest its production arm and license the new technology as an industry outsider in which case it can replicate multiproduct monopoly profits. I derive optimal contracts and optimal number of licenses by assuming a logit demand system. Optimal number of licenses, quite strikingly, increase when the technology has a higher relative value to a commercialized alternative. This result stand in sharp contrast with literature on licensing of a homogenous good.

Keywords: Patent licensing, price competition, product differentiation.

JEL Classification: D45, K11, L11, L13, L21, L41.

Introduction

Technological innovation is probably the most important way we generate value. Innovators, naturally, need to see a profit opportunity, that is, a way to extract some of this value, in order innovate in the first place. Thus, we have a established a number of ways to give the right incentives to innovators by patents, copyrights or trade secrets.

According to Hall, Jaffe, and Tratjenberg (2001), the number of patents issued in the U.S. between 1963 and 1999 is 2,923,922. Wesley, Cohen and Walsh (2000) report between 1983-1995 patent awards grew by 78% in the U.S. based on the Carnegie Mellon Survey on Industrial R & D. Interestingly, there are ten industries in which 40% or more of the respondents report licensing revenue as a motive for patenting. That is, transfer of technology is seen as a source of revenue in some industries.

Quite often, innovation takes place outside of industry, for example in universities. Given that, they lack production possibilities, technology transfer from universities has become a hot topic of debate recently. Jensen and Thursby (2001), based on "The 1996 Survey of the Association of University Technology Managers", reports that licenses executed increased 75% percent between 1991 and 1996, with 13,087 licenses executed over the entire period. Moreover, the respondents of survey have stated licensing revenue as a major goal.

Given that licensing is a common way innovators generate revenue, it is worth briefly to look at licensing practices. Rostoker (1983) presents a survey of corporate licensing. Among many other facts, it is reported that 46% of the time compensation took the form of a downpayment and running royalties, while 13% were paid up licenses and 39 % involved just royalties. Royalty rates change from 0.2% of sales to 35%.

Caves et al. (1983) also report average royalty rates at about 3% of sales. Licenses have many other dimensions concerning exclusivity, operation areas, future access to new technologies and so on. Licensors seem to have only extracted one third to one half of the returns. Uncertainty, moral hazard, natural incompleteness of licensing contracts may be the reason for this seemingly low share of the licensor revenue relative to the total. Moreover, fees and royalties may be determined using imitation, rule of thumb, industry averages etc, as opposed to optimizing behavior.

A more recent survey is by Macho-Stadler et al.(1996) on licensing behavior of Spanish companies. Overall, there seems to be quite a bit of licensing activity. Royalties seem to be used more frequently than expected but two part contracts are also quite common as well.

Thus, licensing is an integral part of technological innovation. Here, licensing means the transfer of the rights to a technology in exchange of monetary compensation. Many other kinds of economic activity are terribly similar in nature. Often, governments limit access to a market by giving operation licenses, such as for airlines, buses, taxi-cabs etc. A very recent example is the distribution of the third generation mobile communication, UMTS, licenses which award the rights to use certain frequency bands for carrying telephony and data traffic. Occasionally governments require foreign firms to pay tariffs, in order to gain access to their national markets. In telecommunications, operators compensate each other for calls terminated on each others' networks. One could even view wholesale-retailer relationships as a kind of licensing, where the manufacturer sells the retailer the right to further market a product.

Therefore, the bilateral relationship between the owner of a right and another willing to use this right is an important kind of economic activity which needs to be studied. I will present a framework using technology licensing as a paradigm to address several important questions related to how and who to license. The literature on such issues is vast, nevertheless, I will argue that the new framework provides us with some new insights.

There may be many different reasons why the exclusive owner of a technology will transfer this right in exchange for monetary compensation. A convenient list is provided by Katz and Shapiro (1985):

- Increasing marginal costs,
- Capacity constraints,
- Expand the scale of use,
- Limited experience in some markets,
- Differentiated products.

Even though, licensing has been studied quite extensively in the Industrial Organization literature, the last item "Differentiated Products" has not received enough attention. The main point of departure in this paper is to study licensing in a differentiated products industry. In most of the paper, this differentiation is modelled to be due to consumer heterogeneity as opposed to product heterogeneity even though it is quite straightforward to account for the latter kind of heterogeneity also.

Before introducing ideas studied in this paper, I present a brief review of the literature below. Katz and Shapiro (1985) shows in a cournot-duopoly model that it may be profitable for minor innovations to be licensed with fixed fee, and with two-part contracts monopoly outcome can be replicated. Any innovation that increases industry profits when licensed will be licensed. There is a series of papers—Kamien and Tauman (1986), Kamien, Oren and Tauman (1992), Katz and Shapiro (1986)— aiming to pinpoint optimal licensing contracts. These papers study the licensing strategies of the exclusive holder of a cost reducing technology in a Cournot oligopoly model. Licensor is an industry outsider, e.g. a research lab. Three types of strategies are considered: auctioning off a number of licenses, fixed fee licensing or royalties.[1] A common result is that licensing with a fee is better than royalty licensing. The market structure critically depends on the nature of

[1] It is clear that any two part contract would be superior than either a fixed fee or a royalty contract, since the latter two are degenerate two part contracts.

the innovation. For all three strategies, the number of firms in the industry will decrease as the innovation becomes more significant. Moreover, drastic innovations are licensed to a single firm.

These papers, however, do not consider the possibility of potential licensee's to undertake costly R & D and develop a competing technology. Gallini (1984) shows that an incumbent firm may license its technology to preempt other firms from innovating. Rockett (1990) considers a similar model and shows that, upon expiration of a patent, a patentee might strategically license the technology to a weak competitor to prolong its dominance afterwards. Yi (1999) present a rather detailed analysis on the use of licensing as an entry or innovation deterrence strategy.

Shepard (1987) studies a model where in equilibrium a firm with the exclusive rights to a new producer good may license it in order to enhance industry demand. Economides (1995) shows that in the presence of strong network externalities the exclusive owner of a new technology may even license it for free to its Cournot competitors in a homogeneous good market. Two excellent surveys on licensing technological innovations are Reinganum (1989) and Kaimen (1990).

There are three recent papers which are closely related to this one. Hernàndez-Murillo and Llobet (2002) study licensing of a cost reducing technology, when there is product differentiation and firms are heterogenous with respect to their production technologies. For the downstream market, they adopt a monopolistic competition framework, where a representative agent consumes all from a continuum of products. They show that a patent holder will always employ a royalty in addition to a fixed fee. A similar result obtains in this paper also, and this is not surprising since a royalty can be used as an instrument to control the marginal costs of final products and thus to control final price levels. They find that the optimal number of licenses will increase as the level of product differentiation decreases.

Sen and Tauman (2002) study licensing in a Cournot oligopoly where the licensor employs a combination of an auction and running royalties as the mechanism for compensation. The mechanism considers a licensor which sets royalty fees and the number of licenses, and then sells these licenses by means of an auction. Moreover, they consider licensing by an industry insider and an outsider as in this paper. The results they find are in line with previous literature studying licensing in a Cournot oligopoly. They find that as a result of the innovation, consumers are better off, firms are worse off and welfare increases.

Sandonis and Fauli-Oller (2003) study a differentiated products duopoly and investigate incentives of a external patentee to merge with an insider firm. They find that this may be optimal only when the cost reducing innovation is not drastic. In the model, they consider a case where the outside option of a potential licensee is not fixed, and depends on the offered licensing contracts. They highlight the differences of incentives for an insider and an external patentee. The external patentee has more instruments while an insider can not commit to a license contract to its production arm. However, when patentee is an industry outsider, a potential licensee's outside option depends on the offered licensing contracts, and inducing this licensee to accept a licensing contract may become costly. For large innovations, any gain from taking outside option for a potential licensee is low, thus in this case, it is optimal to remain an industry outsider. On the other hand, for small innovations they show that the outside option of a potential licensee is high, resulting in being an industry insider a better option.

In this paper, I introduce product differentiation and also consider licensing by an industry insider. I consider two-part licensing contracts which involve a downpayment and running royalties. I assume that a single firm owns the rights to a drastic innovation. There is a large number of potential licensees whose outside option is to stay out or take part in the competitive production of a commercialized alternative. This implies that the value of not being a licensee is zero.[2]

[2] The framework actually allows any fixed level of outside option value.

A first, quite general, result I derive is that the owner of a technology can implement the multiproduct monopoly outcome, which delivers the highest possible level of profits, by means of a two-part licensing contract. More interestingly, I also show that an industry insider prefers to divest its production arm, and become an industry outsider. This result generalizes that of Sandonis and Fauli-Oller (2003) for drastic innovations. This is due to the commitment problem it faces; that is, an industry insider cannot commit to charge itself a royalty that is higher than her marginal cost plus the opportunity cost of not selling through a licensee, and thus implementing the multiproduct monopoly outcome is not possible. This result holds for quite general specifications of demand, and product differentiation can be due to consumer and product heterogeneity.

Next, I introduce a flexible demand model—logit. By means of an outside option, I allow for commercialized alternatives which may account for the already existing technology. Assuming a particular demand system makes it possible to derive the optimal number of licenses as a function of market size, relative attractiveness of the new technology compared to the commercialized alternatives, fixed costs of production, marginal costs and level of product differentiation or heterogeneity of consumer tastes. I also derive optimal licensing contracts as a by product. The optimal number of licenses decreases in fixed costs of production, marginal costs, level of substitutability, while increases in the market size and relative attractiveness of the new technology.

In section 1, I introduce the model and compare licensing by an industry insider and outsider. The logit demand is introduced in section 2, and I first derive optimal licensing contracts given a fixed number of licences and then the optimal number of licenses in section 3. Section 4 concludes.

1 The Model

Consider a monopolist, M, who is the sole owner of a production technology, and has sunk the costs of development. M has also obtained a patent thus replicating the technology is not legally possible. This technology can be used as an input to producing differentiated goods, and there are many firms who is ready to produce given access to the technology. Let us denote M as firm 1 and the other firms as $\{2, \ldots, N^{max}\}$, with $N^{max} \gg 2$. The demand for each firm's product denoted by $m_k(p_1, \ldots, p_{N^{max}})$. When only N firms are active, the demand is given by

$$m_k(p_1, \ldots, p_N) = \lim_{p_{N+1}, \ldots, p_{N^{max}} \to \infty} m_k(p_1, \ldots, p_{N^{max}}) \tag{1}$$

with $N \leq N^{max}$.

M has a marginal cost of c_j^I for transferring the input technology to firm j. Then each firm also incurs a marginal cost c_j^F to produce the final good, for $j = 1, \ldots, N^{max}$. Moreover, each firm has an fixed operating cost of C_j, $j = 1, \ldots, N^{max}$.

In case M licenses its technology to firm j, it uses a contract which is formed by a fixed fee, f_j and per unit royalty r_j. Price competition takes place given the licensing contracts.

M can decide to produce, in which case I refer to it as an insider, or it can remain as an industry outsider, that it chooses not to produce. Let $\mathbf{x}^N = (x_1, \ldots, x_N)$. Then, the profit of firm 1, M, when it produces is given by

$$\begin{aligned}\Pi_1^{Insider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N) &= (p_1 - c_1^I - c_1^F) m_1(\mathbf{p}^N) - C_1 \\ &\quad + \sum_{j=2}^{N} \left[(r_j - c_j^I) m_j(\mathbf{p}^N) + f_j \right],\end{aligned}$$

and when M only licenses, by

$$\Pi_1^{Outsider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N) = \sum_{j=1}^{N} \left[(r_j - c_j^I) m_j(\mathbf{p}^N) + f_j \right].$$

Furthermore, the profit of firm j when it produces is given by

$$\Pi_j(\mathbf{p}^N, r_j, f_j) = (p_j - r_j - c_j^F)m_j(\mathbf{p}^N) - f_j - C_j, \qquad j = 2..N,$$

otherwise it is fixed and normalized to zero without loss of generality. This normalization is not an innocent one. It inherently assumes that the new technology is replacing an old one which is competitively supplied. Thus the innovation in this context is assumed to be drastic.[3]

Given the sequence of events, it is straight forward to derive the decision problem faced by the owner of the technology both when he is an insider or an outsider assuming that downstream firms compete in prices. Existence of a pricing equilibrium requires certain conditions on the demand function. In the remainder of the text, I assume that the demand functions are such that $\Pi_k(\mathbf{p}^N, r_k, f_k)$, $\Pi_1^{Outsider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N)$, and $\Pi_1^{Outsider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N)$ are quasi-concave in the relevant variables. Therefore, first order conditions are sufficient to characterize best responses, and hence, equilibrium.

Below, I show that an industry outsider would prefer to divest its production arm and license the technology as an industry outsider. This result is due to a commitment problem faced by the owner of the technology when he is an industry insider. If the outside option of a potential licensee depended on the licensing contracts this result may not hold. In fact, Sandonis and Fauli-Oller (2003) shows that for minor innovations it may be better to be an industry insider. Nevertheless, also in their linear demand differentiated products duopoly model, a drastic innovation is licensed by an industry outsider.

Proposition 1. *When M licenses at all, it prefers only to license and not to produce an output good.*

Proof: Assume M licenses to N firms. Pricing game takes place given (f_j, r_j), for $j = 2, \ldots, N$. When M is an outsider equilibrium is determined by the simultaneous solution

[3] If the innovation is thought to reduce marginal costs, this assumption becomes even stronger. It requires that even the monopolist price of the new technology is sufficient to drive the rest of the firm out of business.

of

$$p_j = r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N.$$

On the other hand, when M is an insider the equilibrium is determined by the solution of

$$\begin{aligned} p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 2..N, \\ p_1 &= c_1^I + c_1^F - \frac{m_1(\mathbf{p}^N)}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \\ &\quad - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N) \end{aligned}$$

Given these relationships, M selects optimal contracts (f_j, r_j). When it is an outsider the optimal contracts are selected by the solution of

Program 1

$$\max_{(f_j, r_j), j=1..N} \sum_{j=1}^{N} \left[(r_j - c_j^I) m_j(\mathbf{p}^N) + f_j \right]$$

such that

$$\begin{aligned} p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N \\ f_j &\leq \Pi_j(\mathbf{p}^N, r_j, f_j) - C_j, \qquad j = 1, .., N \end{aligned}$$

Substituting the second set of constraints yields

Program 1′ :

$$A = \max_{(r_j), j=1..N} \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F) m_j(\mathbf{p}^N) - C_j \right]$$

such that

$$p_j = r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N$$

When M is an insider on the other hand the optimal contracts are determined by

Program 2

$$\max_{(f_j, r_j), j=2,\ldots,N} \quad (p_1 - c_1^I - c_1^F) m_1(\mathbf{p}^N) - C_1 + \sum_{j=2}^{N} \left[(r_j - c_j^I) m_j(\mathbf{p}^N) + f_j \right],$$

such that

$$\begin{aligned} p_1 &= c_1^I + c_1^F - \frac{m_1(\mathbf{p}^N)}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \\ &\quad - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N) \\ p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_k(\mathbf{p}^N)}, \qquad j = 2, .., N \\ f_j &\leq \Pi_j(\mathbf{p}^N, r_j, f_j) - C_j, \qquad j = 2, .., N \end{aligned}$$

Let

$$r_1 = c_1^I - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N)$$

and also substitute the last set of constraints in Program 2 to yield

Program 2′

$$B = \max_{(r_j), j=1,\ldots,N} \quad \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F) m_j(\mathbf{p}^N) - C_j \right],$$

such that

$$\begin{aligned} p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N \\ r_1 &= c_1^I - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N) \end{aligned}$$

Comparing Program 1′ and Program 2′, it is apparent that we have the same optimization problem with one more constraint in the second one. Thus, the maximum of the second program must be less or equal to the first one. ■

Notice that, this result is general and applies to any kind of differentiated demand system which satisfy (1). The reason behind this result is that, M cannot commit to a transfer price other than its cost of producing the input plus the opportunity cost not selling through its licensee's when it is an industry insider. On the other hand, whenever it is an outsider, it is free to choose this transfer price. Thus the implication is that under two-part licensing contracts, the owner of the technology is going divest itself from its producing arm if it ever finds licensing attractive. Rey and Tirole (1999) report on AT&T's spinning off its manufacturing arm, which now is called Lucent.

If the firms were allowed more sophisticated contracts, even an industry insider can obtain the multiproduct monopoly profits. Maurer and Scotchmer (2004) suggest a licensing contract which involves a downpayment and royalty which decreases in the output of the patentee may solve this commitment problem and obtain monopoly profits. However, such contracts would probably be deemed anti-competitive and will not be allowed by anti-trust agencies. The result I show in Proposition 1 is thus valid only for two-part licensing contracts of the type I considered. Nevertheless, as mentioned in the introduction, a downpayment plus a running royalty type contracts are quite common in practice.

2 Logit Demand

In this section, I will introduce a parsimonious demand system which allows me to answer a few questions with more specificity. Assume that there is a consumer population of size S. Each consumer demands one unit of the produced brands or an outside alternative. Brand j produced by firm j yields the net utility

$$\bar{U}_{ij} = \bar{V}_j - \alpha p_j + \epsilon_{ij},$$

for consumer i and the outside good

$$\bar{U}_0 = V_0 + \epsilon_{i0}.$$

Assume also that ϵ_{ij} is independent for all i and j, and double exponential distributed with unit variance.[4] The price sensitivity is measured by α and as $\alpha \to \infty$, the product differentiation disappears. $\bar{V}_j$ measures the stand alone value of a product which essentially can be viewed as the population mean of the valuation of product j and ϵ_{ij} measures the deviation of each individuals' personal valuation from this mean.

The presence of an outside option allows us to take the existing technologies into account. If the existing technology is mature and not protected by patents, it is natural to expect that it is supplied competitively. Moreover, it is possible to characterize level of the innovation relative to this existing outside option. [5]

Then the expected market share of product i when there are $N + 1$ alternatives in total is given by

$$m_i(\mathbf{p}^N) = \frac{\exp(\bar{V}_i - \alpha p_i)}{\exp(V_0) + \sum_{j=1}^N \exp(\bar{V}_j - \alpha p_j)} \tag{2}$$

$$= \frac{\exp(V_i - \alpha p_i)}{1 + \sum_{j=1}^N \exp(V_j - \alpha p_j)} \tag{3}$$

with $V_j = \bar{V}_j - V_0$ for $j = 1, \ldots, N$. The demand for each good then is given by the product of market share with the market size S, that is $d_i(\mathbf{p}^N) = S m_i(\mathbf{p}^N)$ where $d_i(\cdot)$ represents the total demand for product i. Observe that this demand system satisfies property (1). For a detailed derivation of the expected market shares see Anderson, de Palma and Thisse (1992). Using this demand system, it is possible to solve the licensing problem in closed form which I present in the next section.

[4] This is just a normalization, that is the underlying preferences are the same when V_j and α are measured in units of this variance.

[5] Given the linear form of consumer utilities, a process innovation that reduces costs is isomorphic to a product innovation that increases the stand alone value. This isomorphism is immediately apparent, if one redefines the price of each firm as a mark-up over marginal cost.

3 Optimal Number of Licenses and Contracts

I assume complete information about the potential licensee's costs as well mean product valuations. By proposition 1 we know that if M is going to license, it will not produce. Therefore, it is sufficient to solve the problem for an industry outsider. Optimal licensing in this framework involves choosing the optimal number of licensees, N, as well optimal contracts (f_j, r_j) for $j = 1, \ldots, N$. To achieve this M, can figure out optimal licensing policies given N, and then compare profits for each N to choose the optimal number of licensees. Thus in the next subsection, I provide the optimal licensing contracts for a given N and the optimal number of contracts is derived in the following subsection.

3.1 Optimal Licensing Contracts to N Firms

First, let us look at the problem when the monopolist produces N brands itself. The optimal prices are found by

$$\max_{p_1,\ldots,p_N} \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F) m_j(\mathbf{p}^N) S - C\right]$$

and satisfy

$$p_j = c_j^I + c_j^F + \frac{1}{\alpha(1 - \sum_{i=1}^{N} m_i(\mathbf{p}^N))} \tag{4}$$

that is, all products are sold at the same mark-up over cost. A detailed derivation of this result can be found in Anderson, De Palma and Thisse (1992).[6] Thus, the only reason a multiproduct monopolist to charge different prices is due to cost differences. Even if the costs, and hence the prices, were the same, the sales of each product may differ due to differences in stand alone values, V_j, however.

The highest possible profit in the industry is attained by prices in (4), as these internalize all the cross product effects. The immediate question then is why one should

[6] They derive this result for nested-logit which is a generalization of the approach adopted here, however, the result applies for the logit also.

license at all. It might be that the monopolist cannot credibly sell differentiated goods. Or, the monopolist might be financially constrained and does not have funds to cover the necessary fixed costs. Alternatively, the monopolist might be forced to license by regulation.

Given N, M has to solve Program 1′ which reduces to

$$max_{(r_j),j=1..N} \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F)m_j(\mathbf{p}^N)S - C_j\right] \tag{5}$$

such that

$$p_j = r_j + c_j^F + \frac{1}{\alpha(1 - m_j(\mathbf{p}^N))}, \qquad j = 1, .., N. \tag{6}$$

It is apparent that an industry outsider can attain the profits a multiproduct monopolist can achieve by an appropriate choice of royalty rate. This result is summarized in the next proposition.

Proposition 2. *An industry outsider can obtain the multiproduct monopoly profits by choosing the royalty rate to be*

$$r_j = c_j^I + \frac{\sum_{i \neq j} m_i(\mathbf{p}^N)}{\alpha(1 - m_j(\mathbf{p}^N))(1 - \sum_{i=1}^{N} m_i(\mathbf{p}^N))}, \qquad j = 1, \ldots, N, \tag{7}$$

which leads to equilibrium prices as given in (4). The fixed fees charged to each firm, f_j, is then given by

$$f_j = (p_j - r_j - c_j^F)m_j(\mathbf{p}^N)S - C_j. \tag{8}$$

Proof. Observe that the highest possible level of profits are obtained by the multiproduct monopolist with the prices given (4). Substituting the prices in (4) in (6) yields

$$c_j^I + c_j^F + \frac{1}{\alpha(1 - \sum_{i=1}^{N} m_i(\mathbf{p}^N))} = r_j + c_j^F + \frac{1}{\alpha(1 - m_j(\mathbf{p}^N))}.$$

Solving for r_j provides the necessary royalty rates. ■

Observe that the royalty rates in (7), are firm specific, that is they depend on any marginal costs that might incurred in the process of transferring the technology, as well as the stand value of a particular product through the market shares that will be realized in equilibrium.

One advantage of adopting the logit approach is that, it allows us to characterize the market shares in terms of the model primitives in closed form. In order to achieve this goal, it is necessary to introduce an auxiliary concept, however. Lambert's W function is defined as

$$W(x) = \{z | z \exp(z) = x\} \tag{9}$$

For x real and positive, $W(x)$ is a positive valued, increasing and concave function of x. I refer the interested reader to Corless et. al. (1996) for more detailed information on the Lambert W function. Given this definition, I characterize the equilibrium market shares in terms of the model parameters in the next proposition.

Proposition 3. *Equilibrium market shares are given by*

$$m_j^* = \frac{K_j}{K} \frac{W(Ke^{-1})}{1 + W(Ke^{-1})}, \qquad j = 1, \ldots, N, \tag{10}$$

where $W(\cdot)$ *is the Lambert W function,* $K_j = \exp(V_j - \alpha(c_j^I + c_j^F))$*, and* $K = \sum_{i=1}^{N} K_i$*. Moreover, the market share of firm* j *increases in* V_j*, and decreases in* α*,* c_j^I *and* c_j^F*.*

Proof. See Appendix.

The equilibrium market shares have expected properties. Products which provide higher surplus are the ones with higher market shares. Characterization of the market shares in Proposition 3, allows me further to compute royalty rates, fixed fees and retail prices as well using (7), (8), and (4). However, when the costs and stand alone values are arbitrary, it is not possible to make statements on how these quantities will change as one of the model primitives changes due to the dependence of each on all of the market shares. Naturally, when marginal cost and stand alone value of each potential licensee is different, the question of who to license becomes more involved. The number of optimal licenses will differ for different distribution of these values across potential licensees. Moreover, it is likely that in a more realistic situation the patent owner will have incomplete information at best, thus the implementation of a licensing mechanism requires more complicated arguments than offered so far. I leave this topic for future research and continue with placing a few restrictions on the model.

One special case arises when firms are symmetric which implies a symmetric equilibrium outcome. Hence, I assume that $V_j = V$, $c_j^I = c^I$, $c_j^F = c^F$ and $C_j = C$ for all j in the remainder of the paper for simplicity. The assumption of symmetry is commonly used in the literature, and is also reasonable ex-ante. The product differentiation in this case is only due to consumer heterogeneity and product heterogeneity is ruled out. In this case, $K_j = k = \exp(V - \alpha(c^I + c^F))$ for all j, and $K = Nk$, thus the equilibrium market share in (10) reduces to

$$m^* = \frac{1}{N}\frac{W(Nke^{-1})}{1 + W(Nke^{-1})}. \tag{11}$$

The comparative statics on m^* are the same as in Proposition (10), thus a new technology which is more valuable relative to the existing one will be diffused more in equilibrium.

Proposition 4. *If all the licensees are symmetric, equilibrium retail price of each product is given by*

$$p^* = c^I + c^F + \frac{1}{\alpha(1 - Nm^*)}. \tag{12}$$

which increases in V, c^I, c^F and decreases in α.

Proof. See Appendix.

Equilibrium retail prices have intuitive properties. Symmetry also implies that all the licensees receive the same licensing contract, that is $r_j = r$ and $f_j = f$ for all j. In the next proposition, I characterize the symmetric licensing contract.

Proposition 5. *When all licensees are symmetric, equilibrium royalty rate for each licensee is given by*

$$r^* = c^I + \frac{(N-1)m^*}{\alpha(1 - m^*)(1 - Nm^*)}, \tag{13}$$

which increases in V, c^I and decreases in α, c^F. The equilibrium fixed fee is given by

$$f^* = \frac{m^*}{\alpha(1-m^*)}S - C. \tag{14}$$

Furthermore, f^* increases with V, S and decreases with α, c^I, c^F and C.

Proof. See Appendix.

It is interesting to note that when licensees are incurring higher marginal costs to transform the licensed technology to the final product, royalty rates tend to be lower. They are also lower in those industries where consumers are less heterogeneous in their tastes for products. A new technology, which allows firms to produce more valuable products relative to the commercialized alternatives (outside option), is licensed with a higher royalty rate and fixed fee.

3.2 Optimal Number of Licenses

The equilibrium profit of M when it licenses to N firms is given by

$$\Pi(N) = \frac{m^*}{\alpha(1-Nm^*)}NS - NC$$

thus, if we were to treat N as a continuous variable temporarily, it is easy to verify that

$$\frac{\partial \Pi(N)}{\partial N} = \frac{m^*}{\alpha}S - C, \tag{15}$$

since

$$\frac{\partial m^*}{\partial N} = -m^{*2}(2 - Nm^*). \tag{16}$$

Whenever, (15) is positive at $N = 1$, the owner of the patent will prefer licensing or producing more than one product which I assume in the following.

Furthermore, (16) implies concavity of the profits in N, thus, solving (15) is sufficient for a maximum. Observe, however, that N can take only integer values, therefore optimal number of licenses must be one of the two closest integers to the value of N which satisfy

$$\frac{1}{N}\frac{W(Nke^{-1})}{1+W(Nke^{-1})} = \frac{\alpha C}{S}.$$

The solution is summarized in the next proposition.

Proposition 6. *The optimal number of licenses is given by one of the integers closest to*

$$N^* = \frac{(W(\frac{kS}{\alpha C}) - 1)S}{\alpha W(\frac{kS}{\alpha C})C}. \tag{17}$$

Moreover, N^* increases with V and S, while it decreases with α, c^I, c^F and C.

Proof. See Appendix.

The fact that the optimal number of licenses increase with V, or similarly decrease with c^F, implies that a new technology which is drastically better then alternatives will be licensed to more firms. This result stands in sharp contrast with the homogenous goods Cournot-oligopoly licensing literature. The underlying reason is the value of variety in the present model, that is, when there are more brands, more consumers opt for one of the products rather buying the commercialized alternative—a typical feature for differentiated products.

Empirical evidence for such practices is vast and often referred to as *puzzling*. A good example is the liberal licensing policies adopted by Phillips and Sony in case of Audio CD technology. Even though there may have been additional reasons, such as network effects, increasing CD sales[7], there is no question that CD technology is a large leap from the vinyl-records or audio cassettes for delivering printed music. Moreover, either of these technologies were widely available and produced by many firms at the time of the introduction of the CD technology in early eighties. According Grindley and Mc Bride(1992), there were more than 30 licensees of the CD technology by 1981. Both Phillips and Sony produces CD-players as well, however, considering the size of each company, it might be that the commitment problem they faced was not much of an issue, and therefore they did not divest their production arms.

On the other hand, as α increases the the effects of consumer heterogeneity vanishes, thus products are perceived to be closer substitutes. The optimal number of licenses tend

[7]Both Phillips and Sony owned two of the largest record companies at the time.

to decrease in this case. It is apparent from the (15) that, for sufficiently large α, it may be that only one firm is licensed. This result implies in some sense the continuity of the outcomes in the degree of heterogeneity of consumer tastes. When consumers are homogenous, that is when $\alpha \to \infty$, the optimal number of licenses is one, which would be the case with price competition in a homogenous products market.

4 Conclusions

I analyzed patent licensing when products are differentiated due to consumer and/or product heterogeneity. I considered two part licensing contracts which involves a fixed downpayment and running royalties which are quite common in practice. The first result, which holds quite generally, is that an industry insider prefers to divest its production arm and license the new technology as an industry outsider. This is due to a commitment problem faced by the industry insider, which can only credibly commit to a transfer price(self-royalty) of marginal cost plus opportunity cost of not licensing. While, when the patent owner remains outside of the industry it is free to choose this royalty, and in fact, is able to replicate multiproduct monopoly profits by using two part licensing contracts.

Given that a patent owner is better off as an industry outsider, the licensing problem involves choosing a fixed fee, a royalty rate and number of licenses. I derived optimal contracts given a certain number of licenses, and then obtained optimal number of licenses by assuming a logit demand system. I characterized equilibrium market shares in terms of the model primitives, which in turn allowed me to further characterize equilibrium prices and licensing contracts under a symmetry assumption.

Contracts have intuitive properties, both royalty rate and fixed fee increase with the relative value of the new technology. Both decrease with the marginal cost of transforming the input technology to a final good. Interestingly, any marginal costs, the patent owner might incur due to the transfer of the technology increase royalty rate while decrease the fixed fee. The fixed fee is larger for a larger market. The retail prices increase in the relative value of the technology, while decrease with substitutability of products and marginal costs.

Finally, I derived optimal number of licenses, which quite strikingly increase when the technology has a higher relative value to a commercialized alternative. This result stand in sharp contrast with literature on licensing of a homogenous good. The main force behind this result is the fact that variety has value, that is, more consumers purchase the new technology when there are more products employing it.

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Appendix

Proof of Proposition 3

The equilibrium market shares satisfy

$$\frac{m_j(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} = \exp(V_j - \alpha c_j - \frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)}), \tag{18}$$

where $c_j = c_j^I + c_j^F$. Rearranging (18) yields

$$\frac{m_j(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} \exp(\frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)}) = K_j. \tag{19}$$

Summing (19) over j results

$$\frac{\sum_{i=1}^N m_i(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} \exp(\frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)}) = K. \tag{20}$$

Let

$$y = \frac{\sum_{i=1}^N m_i(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)},$$

then it is easy to verify that

$$\frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} = 1+y.$$

Therefore, (20) can be re-written as

$$y\ e^y = \frac{K}{e}. \tag{21}$$

Thus, $y = W(Ke^{-1})$, and also observe that

$$m_j(\mathbf{p}^N)(1+y)exp(1+y) = K_j.$$

Hence, the equilibrium market share of firm j then is given by (10).

For proving the second part of the proposition, it is first useful to note that

$$\begin{aligned} \frac{\partial}{\partial K_j} m_j^* &= \left[\frac{W(Ke^{-1})}{(1+W(Ke^{-1}))^3}\frac{K_j}{K^2} + \frac{W(Ke^{-1})}{(1+W(Ke^{-1}))}\frac{\sum_{i\neq j} K_i}{K^2}\right] \\ &= \frac{1}{K}\left[(1-\sum_{i=1}^N m_i^*)^2 m_j^* + \sum_{i\neq j}^N m_i^*\right] > 0, \end{aligned} \tag{22}$$

that is, the market share of firm j is increasing in K_j. The rest of the comparative statics directly follow.■

Proof of Proposition 4

Imposing symmetry implies that the equilibrium outcome is also symmetric, and thus the equilibrium market share of each firm is given by

$$m^* = \frac{1}{N}\frac{W(Nke^{-1})}{1+W(Nke^{-1})}.$$

Now a change in one of the model primitives leads to a change in the surplus of all the firms, $log(k)$, consequently it is easy to verify that

$$\frac{\partial}{\partial k}m^* = \frac{1}{k}m^*(1-Nm^*)^2 > 0.$$

Also note that $\partial k/\partial V = k > 0$, $\partial k/\partial c^H = -\alpha k > 0$, for $H \in \{I, F\}$ and $\partial k/\partial \alpha = -(c^I + c^F)k > 0$. Therefore,

$$\frac{\partial}{\partial V}p^* = \frac{N}{\alpha(1-Nm^*)^2}\big(\frac{1}{k}m^*(1-Nm^*)^2\big)k = \frac{Nm^*}{\alpha} > 0,$$

$$\begin{aligned}\frac{\partial}{\partial \alpha}p^* &= -\frac{1}{\alpha^2(1-Nm^*)} + \frac{N}{\alpha(1-Nm^*)^2}\big(\frac{1}{k}m^*(1-Nm^*)^2\big)(-(c^I+c^F)k) \\ &= -\frac{1}{\alpha^2(1-Nm^*)} - \frac{Nm^*(c^I+c^F)}{\alpha} < 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial c^H}p^* &= 1 + \frac{N}{\alpha(1-Nm^*)^2}\big(\frac{1}{k}m^*(1-Nm^*)^2\big)(-\alpha k) \\ &= 1 - Nm^* > 0,\end{aligned}$$

for $H \in \{I, F\}$.■

Proof of Proposition 5 It is easy to verify that the (7) and (8), reduces to (13) and (14) when symmetry is imposed. To derive the comparative statics, first note that

$$\frac{\partial}{\partial m^*}r^* = \frac{(N-1)(1-Nm^{*2})}{\alpha(1-m^*)^2(1-Nm^*)^2} > 0,$$

and

$$\frac{\partial}{\partial m^*} f^* = \frac{S}{\alpha(1-m^*)^2} > 0.$$

Then

$$\frac{\partial}{\partial V} r^* = \frac{\partial r^*}{\partial m^*} \frac{\partial m^*}{\partial k}(k) > 0,$$

$$\frac{\partial}{\partial c^F} r^* = \frac{\partial r^*}{\partial m^*} \frac{\partial m^*}{\partial k}(-\alpha k) < 0$$

$$\begin{aligned}
\frac{\partial}{\partial c^I} r^* &= 1 + \frac{\partial r^*}{\partial m^*} \frac{\partial m^*}{\partial k}(-\alpha k) \\
&= \frac{(N-1)(1-m^*-(N-1)m^{*2})}{(1-m^*)^2} > 0,
\end{aligned}$$

since $1 - m^* - (N-1)m^{*2} > 1 - m^* - (N-1)m^* > 0$, and

$$\frac{\partial}{\partial \alpha} r^* = -\frac{(N-1)m^*}{\alpha^2(1-m^*)(1-Nm^*)} + \frac{\partial r^*}{\partial m^*} \frac{\partial m^*}{\partial k}(-(c^I + c^F)k) < 0.$$

Similarly, the comparative statics can be found found for f^* as

$$\frac{\partial}{\partial V} f^* = \frac{\partial f^*}{\partial m^*} \frac{\partial m^*}{\partial k}(k) > 0,$$

$$\frac{\partial}{\partial S} f^* = \frac{m^*}{\alpha(1-m^*)} > 0,$$

$$\frac{\partial}{\partial C} f^* = -1 < 0,$$

$$\frac{\partial}{\partial c^H} f^* = \frac{\partial f^*}{\partial m^*} \frac{\partial m^*}{\partial k}(-\alpha k) < 0$$

for $H \in \{I, F\}$, and

$$\frac{\partial}{\partial \alpha} f^* = -\frac{m^* S}{\alpha^2(1-m^*)} + \frac{\partial f^*}{\partial m^*} \frac{\partial m^*}{\partial k}(-(c^I + c^F)k) < 0.$$

■

Proof of Proposition 6 Let $L = \alpha C/S$, then it is easy to verify that

$$W(Nke^{-1}) = \frac{NL}{1-NL}.$$

By the definition of the Lambert's W function we have

$$\begin{aligned} \frac{NL}{1-NL}\exp(\frac{NL}{1-NL}) &= Nke^{-1}, \\ \frac{1}{1-NL}\exp(\frac{1}{1-NL}) &= \frac{k}{L}. \end{aligned}$$

Therefore,

$$W(\frac{k}{L}) = \frac{1}{1-NL}, \tag{23}$$

and solving (23) for N yields the expression for N^*. In order to perform the comparative statics, first note that,

$$\frac{\partial}{\partial k}N^* = \frac{1}{W(k/L)(1+W(k/L))Lk} > 0,$$

and

$$\frac{\partial}{\partial L}N^* = -\frac{W(k/L)}{(1+W(k/L))L^2} < 0.$$

Furthermore, $\partial L/\partial S = -\alpha C/S^2$,$\partial L/\partial \alpha = C/S$, and $\partial L/\partial C = \alpha/S$. Therefore,

$$\frac{\partial}{\partial V}N^* = \frac{\partial}{\partial k}N^* k > 0,$$

$$\begin{aligned} \frac{\partial}{\partial S}N^* &= \frac{\partial}{\partial L}N^*(-\frac{\alpha C}{S^2}) > 0, \\ \frac{\partial}{\partial C}N^* &= \frac{\partial}{\partial L}N^*(\frac{\alpha}{S}) < 0, \\ \frac{\partial}{\partial c^H}N^* &= \frac{\partial}{\partial k}N^*(-\alpha k) < 0, \end{aligned}$$

for $H \in \{I, F\}$, and

$$\frac{\partial}{\partial \alpha}N^* = \frac{\partial}{\partial k}N^*(-(c^I + c^F)k) + \frac{\partial}{\partial L}N^*(\frac{C}{S}) < 0.$$

■